

NAG Toolbox for MATLAB

f07pe

1 Purpose

f07pe solves a real symmetric indefinite system of linear equations with multiple right-hand sides,

$$AX = B,$$

where A has been factorized by f07pd, using packed storage.

2 Syntax

```
[b, info] = f07pe(uplo, ap, ipiv, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07pe is used to solve a real symmetric indefinite system of linear equations $AX = B$, the function must be preceded by a call to f07pd which computes the Bunch–Kaufman factorization of A , using packed storage.

If **uplo** = 'U', $A = PUDU^T P^T$, where P is a permutation matrix, U is an upper triangular matrix and D is a symmetric block diagonal matrix with 1 by 1 and 2 by 2 blocks; the solution X is computed by solving $PUDY = B$ and then $U^T P^T X = Y$.

If **uplo** = 'L', $A = PLDL^T P^T$, where L is a lower triangular matrix; the solution X is computed by solving $PLDY = B$ and then $L^T P^T X = Y$.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – string

Indicates how A has been factorized.

uplo = 'U'

$A = PUDU^T P^T$, where U is upper triangular.

uplo = 'L'

$A = PLDL^T P^T$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **ap**(*) – double array

Note: the dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n} + 1)/2)$.

The factorization of A stored in packed form, as returned by f07pd.

3: **ipiv**(*) – int32 array

Note: the dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

Details of the interchanges and the block structure of D , as returned by f07pd.

4: **b(ldb,*)** – double array

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs_p})$

The n by r right-hand side matrix B .

5.2 Optional Input Parameters1: **n** – int32 scalar

Default: The first dimension of the array **ap** and the second dimension of the array **ap**. (An error is raised if these dimensions are not equal.)

n , the order of the matrix A .

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p** – int32 scalar

Default: The second dimension of the array **b**.

r , the number of right-hand sides.

Constraint: $\mathbf{nrhs_p} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldb

5.4 Output Parameters1: **b(ldb,*)** – double array

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs_p})$

The n by r solution matrix X .

2: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **nrhs_p**, 4: **ap**, 5: **ipiv**, 6: **b**, 7: **ldb**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

For each right-hand side vector b , the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

if **uplo** = 'U', $|E| \leq c(n)\epsilon P|U||D||U^T|P^T$;

if **uplo** = 'L', $|E| \leq c(n)\epsilon P|L||D||L^T|P^T$,

$c(n)$ is a modest linear function of n , and ϵ is the *machine precision*.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_{\infty}}{\|x\|_{\infty}} \leq c(n) \operatorname{cond}(A, x) \epsilon$$

where $\operatorname{cond}(A, x) = \| |A^{-1}| |A| |x| \|_{\infty} / \|x\|_{\infty} \leq \operatorname{cond}(A) = \| |A^{-1}| |A| \|_{\infty} \leq \kappa_{\infty}(A)$.

Note that $\operatorname{cond}(A, x)$ can be much smaller than $\operatorname{cond}(A)$.

Forward and backward error bounds can be computed by calling f07ph, and an estimate for $\kappa_{\infty}(A)$ ($= \kappa_1(A)$) can be obtained by calling f07pg.

8 Further Comments

The total number of floating-point operations is approximately $2n^2r$.

This function may be followed by a call to f07ph to refine the solution and return an error estimate.

The complex analogues of this function are f07ps for Hermitian matrices and f07qs for symmetric matrices.

9 Example

```
uplo = 'L';
ap = [2.07;
      4.2;
      0.2230413840558341;
      0.6536583767489105;
      1.15;
      0.8115010321439103;
      -0.5959697237786296;
      -2.59067708640519;
      0.3030846795506181;
      0.4073851981348882];
ipiv = [int32(-3);
        int32(-3);
        int32(3);
        int32(4)];
b = [-9.5, 27.85;
      -8.380000000000001, 9.9;
      -6.07, 19.25;
      -0.96, 3.93];
[bOut, info] = f07pe(uplo, ap, ipiv, b)
```

```
bOut =
    -4.0000    1.0000
    -1.0000    4.0000
     2.0000    3.0000
     5.0000    2.0000
info =
      0
```